

Neutrino decay as a possible interpretation to the MiniBooNE observation with unparticle scenario

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Abstract

In a new measurement on neutrino oscillation $\nu_\mu \rightarrow \nu_e$, the MiniBooNE Collaboration observes an excess of electron-like events at low energy and the phenomenon may demand an explanation which obviously is beyond the oscillation picture. We propose that heavier neutrino ν_2 decaying into a lighter one ν_1 via the transition process $\nu_\mu \rightarrow \nu_e + X$ where X denotes any light products, could be a natural mechanism. The theoretical model we employ here is the unparticle scenario established by Georgi. We have studied two particular modes $\nu_\mu \rightarrow \nu_e + \mathcal{U}$ and $\nu_\mu \rightarrow \nu_e + \bar{\nu}_e + \nu_e$. Unfortunately, the number coming out from the computation is too small to explain the observation. Moreover, our results are consistent with the cosmology constraint on the neutrino lifetime and the theoretical estimation made by other groups, therefore we can conclude that even though neutrino decay seems plausible in this case, it indeed cannot be the source of the peak at lower energy observed by the MiniBooNE collaboration and there should be other mechanisms responsible for the phenomenon.

I. INTRODUCTION

Recently, the MiniBooNE Collaboration reported its results of searching for $\nu_\mu \rightarrow \nu_e$ oscillations [1]. In the experiment, the ν_μ energy spectrum has a peak centered at 700 MeV and extends to 3000 MeV. For the oscillation range $475 < E_\nu < 1250$ MeV where E_ν is the energy of the produced neutrino, no significant excess of events is found. This result excludes sizable appearance of ν_e via two neutrino oscillation and disfavors the previous LSND measurement [2]. However, they observed that outside of the oscillation range, there is a clear peak of the electron-neutrino-like events ($96 \pm 17 \pm 20$ events) lying above background at $300 < E_\nu < 475$ MeV. Although the origin of the excess is still under investigation, we may assume that they are indeed electron neutrinos at present. The beam is completely composed of ν_μ and the oscillation can only produce ν_e with the same energy, therefore the observation would be a serious challenge to the present theories. Namely a reasonable explanation about the appearance of the low energy ν_e is needed. To answer this question, there are some interesting proposals, for example, in [3] the authors suggest a (3+2) neutrino oscillation scenario where two sterile neutrinos are introduced into the game to explain the MiniBooNE results and Bodek[4] considered the internal bremsstrahlung as an alternative source of the excess ν_e events. Instead, in this work, we are looking for possible mechanisms other than the neutrino oscillation, supposing that there are only standard model (SM) neutrinos. An explanation that ν_μ may decay into $\nu_e + X$ where X denotes some possible light products, seems reasonable. Definitely, neutrino decay must be realized via interactions beyond the SM. The possible candidates of X could be $\nu_e + \bar{\nu}_e$, light bosons (for example axion etc.) and the unparticle which we are going to explore in this work.

In fact, the idea of neutrino decay is not new. It has been put forward by some authors [5, 6]. The basic idea is to introduce a heavy, unstable neutrino (usually assuming a sterile one) which decays into light neutrino or antineutrino plus a scalar particle. The interactions between the scalar particle and neutrinos are described by a lepton flavor violating effective lagrangian which depends on the details of various new physics models. Instead, we suggest an alternative scenario, namely the heavier ν_2 which is a mass eigenstate and a component of the flavor eigenstate ν_μ , is the constituents of the beam and decays into a light neutrino ν_1 and a scale-invariant unparticle proposed recently by Georgi [7].

It is well known that at very high energy scale, the unparticle physics contains the SM fields and a sector of Banks-Zaks field (defined in [8]) with a non-trivial infrared fix point. Below an energy scale Λ_U which is of order of TeV, the Banks-Zaks fields are matched onto a scale invariant unparticle sector. The unparticle is different from the ordinary particles as it has no mass since the mass term breaks the scale invariance, but the Lorentz-invariant four-momentum square needs not to be zero, $P^2 \geq 0$. The scale dimension of unparticle is in general fractional rather than an integral number (the dimension for a fermion is half-integers). This special characteristic brings us a natural explanation of the shape of low

energy ν_e bump observed by the MiniBooNE Collaboration. If ν_2 decays into a ν_1 and a real scalar particle where ν_i are neutrino mass eigenstates, it is a two-body decay where the energy-spectrum of the produced ν_e should be discrete. It is in contrary to the observation where the energy spectrum of the produced ν_e is continuous. Indeed, the incident ν_μ beam has an energy distribution which can result in a natural energy spreading for the produced electron-neutrino, however, it demands that the shape of ν_e spectrum must be similar to that of the incident ν_μ beam. Instead, if the produced X is an unparticle, the energy spectrum of ν_e would naturally spread and it may be more consistent with the present measurements. The interactions between the unparticle and the SM particles are described in the framework of low energy effective theory and lead to various interesting phenomenology. There have been many phenomenological explorations on possible observable effects caused by unparticles [7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40] and much more are coming up.

The MiniBooNE results indicate that the energy of the events of excess is about a half of the peak position at the energy spectrum of the muon neutrino. As discussed above, we suggest a decay mode $\nu_2 \rightarrow \nu_1 + \mathcal{U}$ where \mathcal{U} denotes the unparticle and a consequent transition $\nu_\mu \rightarrow \nu_e + \mathcal{U}$ might be observed, namely ν_μ and ν_e are not physical eigenstates, but are that of weak interaction and can be caught by detector as an appearance of ν_e at lower energy. As indicated in [7], the unparticle stuff with scale dimension $d_{\mathcal{U}}$ cannot be "seen" directly, it would manifest itself as a missing energy. When the scale dimension $d_{\mathcal{U}}$ is not very large, the energy spectrum of electron neutrino can fall into the allowed range of the MiniBooNE measurements. This process has also been considered in [26]. A transition into a three-body final state $\nu_\mu \rightarrow \nu_e + \nu_e + \bar{\nu}_e$ is also a possible process to explain the MiniBooNE data. The two decay modes: $\nu_\mu \rightarrow \nu_e + \mathcal{U}$ and $\nu_\mu \rightarrow \nu_e + \nu_e + \bar{\nu}_e$ are both lepton flavor violating processes and can only occur via new physics mechanism beyond the SM.

If the mechanism proposed above can explain the observed peak which depends on its decay width, the possible detection rate of the number of ν_e is roughly

$$N_{\nu_e} \sim N_0 (1 - e^{-t/\tau}) \times \eta, \quad (1)$$

where N_0 is the muon neutrino number, τ is the lifetime which should be calculated in the aforementioned scenario and η is a detection rate which is also very small, say, 10^{-10} or even smaller. t is the flight time from the source to the detector and since the speed of the beam neutrino is very close to the speed of light, $t \sim L/c$ where L is the distance and approximately 500 m in the MiniBooNE experiments. In addition the ratio would be further suppressed by the time dilation factor $\gamma = m/E$. Since L is only of several hundred meters, to make a sizable ratio which can be observed, τ must not be large. We will obtain its value by imputing all the concerned model parameters which are fixed by fitting data of other experiments into our numerical computation.

As is well-known, neutrino oscillation had been observed in the solar, atmospheric, accel-

erator neutrino experiments and the present theoretical studies almost completely confirm the MSW mechanism. The relevant mixing parameters and the mass-square differences are determined by fitting the data, even though the absolute values of the neutrino masses are still not fixed yet. While theoretically determining the parameters, possible neutrino decays are not taken into account seriously. How to reconcile the neutrino decay with the present theoretical works on the neutrino oscillation is an open question, namely, one should explore if there exists discrepancy between the theoretical predictions and data. Obviously if the decay rate is sufficiently small, one does not need to modify the present theoretical framework about the neutrino oscillation, but if the decay rate is not too small, the data fitting should be re-considered, therefore the MiniBooNE result indeed provides a new challenge to the neutrino physics and we will return to this topic in our next work [41].

II. NEUTRINO DECAYS IN UNPARTICLE PHYSICS

We start with a brief review of the unparticle physics. First let us consider an unparticle \mathcal{U} with scale dimension $d_{\mathcal{U}}$ and momentum P . The unparticle momentum satisfies the constraint $P^2 \geq 0$. According to [18], the unparticle stuff can be viewed as a tower of massive particles with mass spacing tends to zero. Scale invariance provides the most important constraint on the properties of unparticles. The two-point function of scalar unparticle field operator $O_{\mathcal{U}}$ is written as

$$\langle 0|O_{\mathcal{U}}(x)O_{\mathcal{U}}^\dagger(0)|0\rangle = \int \frac{d^4P}{(2\pi)^4} e^{-iP \cdot x} |\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2 \rho(P^2), \quad (2)$$

where $|P\rangle$ is the unparticle state with momentum P and the phase space factor is

$$|\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2 \rho(P^2) = A_{d_{\mathcal{U}}} \theta(P^0) \theta(P^2) (P^2)^{d_{\mathcal{U}}-2}, \quad (3)$$

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \quad (4)$$

For the vector unparticle field $O_{\mathcal{U}}^\mu$, we have

$$\langle 0|O_{\mathcal{U}}^\mu(0)|P\rangle \langle P|O_{\mathcal{U}}^\nu(0)|0\rangle \rho(P^2) = A_{d_{\mathcal{U}}} \theta(P^0) \theta(P^2) (-g^{\mu\nu} + P^\mu P^\nu / P^2) (P^2)^{d_{\mathcal{U}}-2}, \quad (5)$$

where the transverse condition $\partial_\mu O_{\mathcal{U}}^\mu = 0$ is required. The Lorentz structure of unparticle can also be tensor [10, 32] or spinor [11]. In this study, we restrict our discussions to scalar and vector unparticles. Obviously similar analysis can be done for the tensor and spinor unparticles.

About the virtual effects, the propagator of the scalar unparticle field is given as

$$\int d^4x e^{iP \cdot x} \langle 0|TO_{\mathcal{U}}(x)O_{\mathcal{U}}(0)|0\rangle = i \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{1}{(P^2 + i\epsilon)^{2-d_{\mathcal{U}}}} e^{-i(d_{\mathcal{U}}-2)\pi}, \quad (6)$$

and for the vector unparticle field, the propagator is

$$\int d^4x e^{iP \cdot x} \langle 0 | T O_{\mathcal{U}}^\mu(x) O_{\mathcal{U}}^\nu(0) | 0 \rangle = i \frac{A_{d_{\mathcal{U}}}}{2 \sin(d_{\mathcal{U}} \pi)} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{(P^2 + i\epsilon)^{2-d_{\mathcal{U}}}} e^{-i(d_{\mathcal{U}}-2)\pi}. \quad (7)$$

The function $\sin(d_{\mathcal{U}}\pi)$ at the denominator implies that the scale dimension $d_{\mathcal{U}}$ cannot be integers for $d_{\mathcal{U}} > 1$ in order to avoid singularity. The phase factor $e^{-i(d_{\mathcal{U}}-2)\pi}$ provides a CP conserving phase which produces peculiar interference effects in high energy scattering processes [9, 10, 32] and CP violation in B decays [12, 28].

In this study, we will discuss interactions between the unparticles and neutrinos. The framework which describes these interactions is a low energy effective theory. For our purpose, the coupling of unparticle to neutrinos (ν_μ and ν_e) is given in the form as

$$\mathcal{L}_{eff} = \frac{c_S^{\nu_\alpha \nu_\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{\nu}_\beta \gamma_\mu (1 - \gamma_5) \nu_\alpha \partial^\mu O_{\mathcal{U}} + \frac{c_V^{\nu_\alpha \nu_\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\nu}_\beta \gamma_\mu (1 - \gamma_5) \nu_\alpha O_{\mathcal{U}}^\mu + h.c.. \quad (8)$$

Here, we have used the $V - A$ type current as in the SM. The c_S and c_V are dimensionless coefficients. The ν_α and ν_β are weak eigenstates with different flavor numbers α and β .

As in [26], the neutrino decay is conveniently represented in the basis of mass eigenstates ν_i ($i=1,2$, we only consider two generations in this case). The interactions between unparticle and neutrinos are rewritten by

$$\frac{c_S^{\nu_i \nu_j}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{\nu}_j \gamma_\mu (1 - \gamma_5) \nu_i \partial^\mu O_{\mathcal{U}} + \frac{c_V^{\nu_i \nu_j}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\nu}_j \gamma_\mu (1 - \gamma_5) \nu_i O_{\mathcal{U}}^\mu + h.c.. \quad (9)$$

The relation between the coupling coefficients $c_{S(V)}^{\nu_\alpha \nu_\beta}$ and $c_{S(V)}^{\nu_i \nu_j}$ can be obtained from neutrino mixing matrix. For a simple case, considering two neutrino mixing,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad (10)$$

The coefficients in the mass basis are related to those in the flavor basis by

$$c_{S(V)}^{\nu_1 \nu_2} = \cos^2\theta \, c_{S(V)}^{\nu_\alpha \nu_\beta}. \quad (11)$$

For a maximal mixing where $\theta = \pi/4$, $c_{S(V)}^{\nu_1 \nu_2} = \frac{1}{2} c_{S(V)}^{\nu_\alpha \nu_\beta}$. The coefficients in the different basis differ by a constant factor.

A. The decay of $\nu_2 \rightarrow \nu_1 + \mathcal{U}$

Assuming two generation neutrinos and the heavier one is ν_2 and lighter is ν_1 , the decay of $\nu_\mu \rightarrow \nu_e + \mathcal{U}$ is realized via the transition $\nu_2 \rightarrow \nu_1 + \mathcal{U}$ which is a typical lepton flavor violating process and its Feynman diagram is depicted in Fig. 1. Here, it is natural to assume

that the basis of the interaction between unparticle and neutrino is the same as the weak interaction, namely ν_e and ν_μ are the eigenstates of the interaction. The final unparticle is invisible and behaves as a missing energy. The decay $\nu_2 \rightarrow \nu_1 + \mathcal{U}$ seems to be a two-body process. But it is different from the common case with two final particles whose momenta are single-valued and fixed. For the unparticle case, the energy of ν_1 depends on the momentum square of unparticle P^2 which only is constrained by the condition $P^2 \geq 0$. Namely, one can expect that P^2 can vary within a range $0 \leq P^2 \leq P_{max}^2$ where P_{max} would be determined by the momentum conservation in ν_μ decay. Thus the varying P^2 causes a continuous energy spectrum of E_{ν_e} and it is a characteristic effect of the unparticle.

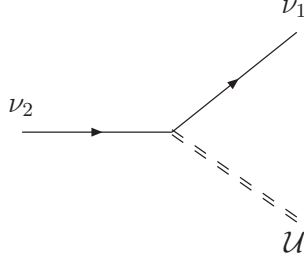


FIG. 1: The diagram for the decay of $\nu_2 \rightarrow \nu_1 + \mathcal{U}$. The double dashed lines represent the unparticle.

In order to make the process realizable, the mass of ν_2 should be larger than that of ν_1 , that is the so-called normal order in literature. Without losing generality, we further assume $m_{\nu_2} \gg m_{\nu_1}$ and neglect m_{ν_1} in the analysis below. The differential decay rate of $\nu_2(p_2) \rightarrow \nu_1(p_1) + \mathcal{U}(q)$ is

$$d\Gamma = \frac{1}{2E_{\nu_2}} \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 d\Phi(p), \quad (12)$$

where the phase space factor $d\Phi(p)$ is

$$d\Phi(p) = (2\pi)^4 \delta^4(p_2 - p_1 - q) \left[2\pi \theta(p_1^0) \delta(p_1^2) \frac{d^4 p_1}{(2\pi)^4} \right] \left[A_{d\mathcal{U}} \theta(q^0) \theta(q^2) (q^2)^{d_{\mathcal{U}}-2} \frac{d^4 q}{(2\pi)^4} \right], \quad (13)$$

with $q = p_2 - p_1$ and the Lorentz-invariant amplitude square is

$$\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{|c_S^{\nu_1 \nu_2}|^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}} 4 p_2^2 (p_1 \cdot p_2), \quad (14)$$

for the scalar unparticle and

$$\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{|c_V^{\nu_1 \nu_2}|^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}} 4 \left[2(p_1 \cdot p_2) + \frac{p_2^2 (p_1 \cdot p_2)}{q^2} \right]. \quad (15)$$

for the vector unparticle.

In the rest frame of ν_2 , it is straightforward to derive the differential decay rate over the ν_1 energy E_1 and the decay rate of $\nu_2(p_2) \rightarrow \nu_1(p_1) + \mathcal{U}(q)$ as

$$\begin{aligned} \frac{d\Gamma_S}{dE_1} &= \frac{|c_S^{\nu_1\nu_2}|^2 A_{d_U}}{2\pi^2 \Lambda_U^{2d_U}} \frac{m_{\nu_2}^2 E_1^2}{(m_{\nu_2}^2 - 2m_{\nu_2} E_1)^{2-d_U}} \theta(m_{\nu_2} - 2E_1), \\ \Gamma_S &= \frac{|c_S^{\nu_2\nu_1}|^2 A_{d_U}}{8\pi^2 d_U (d_U^2 - 1)} m_{\nu_2} \left(\frac{m_{\nu_2}}{\Lambda_U} \right)^{2d_U}, \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{d\Gamma_V}{dE_1} &= \frac{|c_V^{\nu_1\nu_2}|^2 A_{d_U}}{2\pi^2 \Lambda_U^{2d_U-2}} \frac{m_{\nu_2}^2 E_1^2 \left[1 + 2 \left(1 - 2 \frac{E_1}{m_{\nu_2}} \right) \right]}{(m_{\nu_2}^2 - 2m_{\nu_2} E_1)^{3-d_U}} \theta(m_{\nu_2} - 2E_1), \\ \Gamma_V &= \frac{3|c_V^{\nu_1\nu_2}|^2 A_{d_U}}{8\pi^2 d_U (d_U + 1)(d_U - 2)} m_{\nu_2} \left(\frac{m_{\nu_2}}{\Lambda_U} \right)^{2d_U-2}. \end{aligned} \quad (17)$$

with $d_U > 1$ for the scalar unparticle and $d_U > 2$ for the vector unparticle.

In this study, we concern the observation in the laboratory frame where the initial muon neutrino moves nearly with the speed of light. The energy of ν_2 is at the order of several hundred MeV and is much bigger than its inertial mass, $E_2 \gg m_{\nu_2}$. The invariant amplitude square in the laboratory frame depends on the angle θ' between the moving directions of muon and electron neutrinos. Indeed, we need to do some treatments to get an analytical result, i.e. boost the result in the rest frame of ν_2 into the laboratory frame. The momentum of ν_2 is approximated by $|\vec{p}_2| = \sqrt{E_2^2 - m_{\nu_2}^2} \cong E_2(1 - \frac{m_{\nu_2}^2}{2E_2^2})$ and the momentum product $p_2 \cdot p_1 \cong E_1 E_2 (\frac{m_{\nu_2}^2}{2E_2^2} + 1 - \cos\theta')$. From $q^2 \geq 0$, the range of $\cos\theta'$ is determined to be $0 \leq 1 - \cos\theta' \leq \frac{m_{\nu_2}^2}{2E_2 E_1} (1 - \frac{E_2}{E_1})$ which means that three-momentum of the produced electron neutrino is almost parallel to that of the muon neutrino. After performing an integration over $\cos\theta'$, we obtain the differential decay rate of $\nu_2(p_2) \rightarrow \nu_1(p_1) + \mathcal{U}(q)$ in the laboratory frame as

$$\frac{d\Gamma_S}{dE_1} = \frac{|c_S^{\nu_1\nu_2}|^2 A_{d_U}}{4\pi^2} \left(\frac{m_{\nu_2}^2}{\Lambda_U^2} \right)^{d_U} \frac{m_{\nu_2}^2 [1 + y(d_U - 1)] (1 - y)^{d_U-1}}{E_\mu^2 d_U (d_U - 1)} \theta(E_2 - E_1), \quad (18)$$

for the scalar unparticle with $y \equiv \frac{E_1}{E_2}$ and $d_U > 1$. For the vector unparticle, the differential decay rate is

$$\begin{aligned} \frac{d\Gamma_V}{dE_1} &= \frac{|c_V^{\nu_1\nu_2}|^2 A_{d_U}}{16\pi^2} \left(\frac{m_{\nu_2}^2}{\Lambda_U^2} \right)^{d_U-1} \frac{m_{\nu_2}^2}{E_2^2} (1 - y)^{d_U-2} \Gamma(d_U + 1) \\ &\times \left[y(3 - 2y) \frac{{}_2F_1(1, 3; d_U + 2; 1)}{\Gamma(d_U + 2)} + (3 - 7y + 4y^2) \frac{{}_2F_1(2, 3; d_U + 3; 1)}{\Gamma(d_U + 3)} \right. \\ &\quad \left. - 4(1 - y)^2 \frac{{}_2F_1(3, 3; d_U + 4; 1)}{\Gamma(d_U + 4)} \right] \theta(E_2 - E_1). \end{aligned} \quad (19)$$

with $d_U > 2$ and ${}_2F_1(a, b; c; z)$ is the hypergeometric function. The decay rates Γ_S and Γ_V can be obtained and the final results differ from Eqs. (16) and (17) by a Lorentz factor $\frac{m_{\nu_2}}{E_2}$.

B. The three-body decay of $\nu_2 \rightarrow \nu_1 + \bar{\nu}_1 + \nu_1$

As briefly discussed in the introduction, there is another possibility to observe a continuous energy spectrum of ν_1 . Now, we turn to the three-body decay of ν_2 in the framework of unparticle: $\nu_2 \rightarrow \nu_1 + \bar{\nu}_1 + \nu_1$. The Feynman diagram is depicted in Fig. 2 where the unparticle serves as an intermediate agent. Because the final states have two electron neutrinos, there are two diagrams in the process and one needs to consider the anti-symmetrization of the two identical fermions. We consider only the vector unparticle part because the scalar unparticle contribution is proportional to the light neutrino mass and should be very suppressed. According to the effective interaction of neutrinos and unparticle, the decay amplitude of $\nu_2(p_0) \rightarrow \nu_1(p_1) + \nu_1(p_2) + \bar{\nu}_1(p_3)$ is

$$\begin{aligned}\mathcal{M} &= \mathcal{M}_1 + \mathcal{M}_2, \\ \mathcal{M}_1 &= -\frac{c_V^{\nu_1\nu_2} c_V^{\nu_1\nu_1}}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}} \frac{A_{d_{\mathcal{U}}} e^{-i\phi}}{2\sin d_{\mathcal{U}}\pi} \frac{\bar{u}(p_1)\gamma_\mu(1-\gamma_5)u(p_0)\bar{u}(p_2)\gamma_\mu(1-\gamma_5)v(p_3)}{(q_1^2)^{2-2d_{\mathcal{U}}}}, \\ \mathcal{M}_2 &= +\frac{c_V^{\nu_1\nu_2} c_V^{\nu_1\nu_1}}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}} \frac{A_{d_{\mathcal{U}}} e^{-i\phi}}{2\sin d_{\mathcal{U}}\pi} \frac{\bar{u}(p_2)\gamma_\mu(1-\gamma_5)u(p_0)\bar{u}(p_1)\gamma_\mu(1-\gamma_5)v(p_3)}{(q_2^2)^{2-2d_{\mathcal{U}}}},\end{aligned}\quad (20)$$

where $\phi = (d_{\mathcal{U}} - 2)\pi$, $q_1 = p_0 - p_1$ and $q_2 = p_0 - p_2$. The amplitudes \mathcal{M}_1 and \mathcal{M}_2 represent contributions from Fig. 2(a) and (b), respectively. In the derivations, we have neglected $q_1^\mu q_1^\nu / q_1^2$ and $q_2^\mu q_2^\nu / q_2^2$ terms. The square of the invariant matrix element is

$$\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{|c_V^{\nu_1\nu_2} c_V^{\nu_1\nu_1}|^2}{8\Lambda_{\mathcal{U}}^{4d_{\mathcal{U}}-4}} \frac{A_{d_{\mathcal{U}}}^2}{4(\sin d_{\mathcal{U}}\pi)^2} \left[\frac{1}{(q_1^2)^{2-2d_{\mathcal{U}}}} + \frac{1}{(q_2^2)^{2-2d_{\mathcal{U}}}} \right]^2 256(p_0 \cdot p_3)(p_1 \cdot p_2). \quad (21)$$

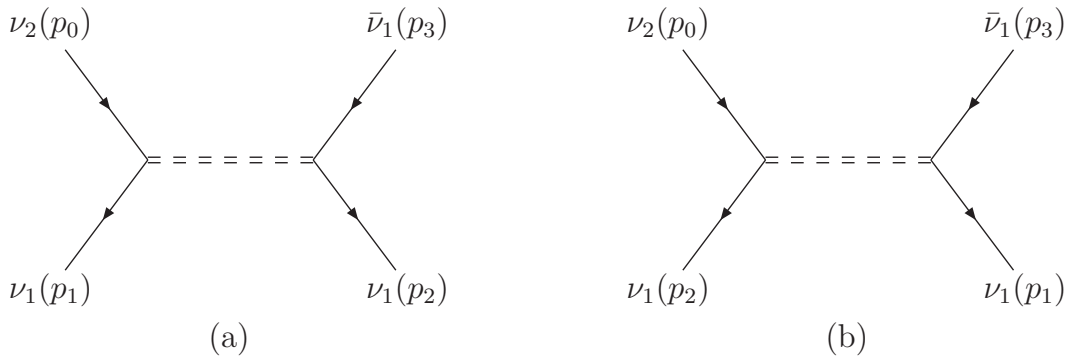


FIG. 2: The Feynman diagram for the three-body decay of $\nu_2 \rightarrow \nu_1 + \nu_1 + \bar{\nu}_1$.

For the three-body decays, we work in the rest frame of the muon neutrino. As will be shown later, the three-body decay rate of $\nu_2 \rightarrow \nu_1 + \bar{\nu}_1 + \nu_1$ is much smaller than that of the two-body decay $\nu_2 \rightarrow \nu_1 + \mathcal{U}$. This conclusion does not depend on which reference frame we

choose. In the rest frame of ν_2 , the three-body kinematics are described in terms of the final neutrino energies E_1, E_2 by

$$\begin{aligned} p_0 \cdot p_3 &= m_{\nu_\mu} (m_{\nu_\mu} - E_1 - E_2), & p_1 \cdot p_2 &= m_{\nu_\mu} \left(E_1 + E_2 - \frac{m_{\nu_\mu}}{2} \right), \\ q_1^2 &= (p_0 - p_1)^2 = m_{\nu_\mu}^2 - 2m_{\nu_\mu} E_1, & q_2^2 &= (p_0 - p_2)^2 = m_{\nu_\mu}^2 - 2m_{\nu_\mu} E_2, \end{aligned} \quad (22)$$

Thus, the differential decay rate is

$$d\Gamma_V(\nu_2 \rightarrow \nu_1 + \nu_1 + \bar{\nu}_1) = \frac{1}{(2\pi)^3} \frac{1}{8m_{\nu_2}} \frac{1}{2} \frac{1}{2} |\mathcal{M}|^2 dE_1 dE_2. \quad (23)$$

where the integration range is $0 \leq E_1 \leq \frac{m_{\nu_2}}{2}$ and $\frac{m_{\nu_2}}{2} - E_1 \leq E_2 \leq \frac{m_{\nu_2}}{2}$.

Note that the similar lepton flavor violating processes $\mu^- \rightarrow e^- + \mathcal{U}$, $\mu^- \rightarrow e^- + e^- + e^+$ have been studied in [15, 22] and their formulations are quite similar to ours.

III. ANALYSIS ON THE CONCERNED PHENOMENOLOGY

For the neutrino accelerator experiment, the neutrinos fly over a baseline with distance L before reaching at the final detectors. If neutrino decays as we suggested, the number of final electron neutrinos produced from muon neutrino decaying is

$$N_{\nu_e} = N_0 \exp \left[1 - \left(-\frac{t}{\tau_{\text{lab}}} \right) \right] \approx N_0 \frac{L}{c} \frac{m}{\tau_\nu E}. \quad (24)$$

with N_0 the initial muon neutrino number, τ_{lab} and τ_ν are neutrino life times in the laboratory and rest frame, respectively. In the MiniBooNE measurement, $L/E \sim 500 \text{ m}/500 \text{ MeV}$, $N_{\nu_e} \sim 100$. The ratio of neutrino life time over mass $\tau_\nu/m_\nu \sim \frac{N_0}{N_{\nu_e}} 10^{-14} \text{ s/eV}$. When $N_0/N_{\nu_e} \sim 10^{10}$, $\tau_\nu/m_\nu \sim 10^{-4}$, and when $N_0/N_{\nu_e} \sim 10^5$, $\tau_\nu/m_\nu \sim 10^{-9}$.

At present, our knowledge on the neutrino mass is mainly obtained from the neutrino oscillation data. The squared mass difference of the mass eigenstates $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ is observed to be [42]: $\Delta m^2 \sim 8 \times 10^{-5} \text{ eV}^2$, $\Delta m^2 \sim 3 \times 10^{-3} \text{ eV}^2$. We will not use the LSND result ($\Delta m^2 \sim 1 \text{ eV}^2$) since it is disfavored by other neutrino experiments and the new MiniBooNE measurements. From these data, we choose $m_2 = 50 \text{ meV}$ as the upper limit in our numerical calculations.

Firstly, we give an observation that the rate of three-body decay $\nu_\mu \rightarrow \nu_e + \bar{\nu}_e + \nu_e$ is much smaller than two-body process $\nu_\mu \rightarrow \nu_e + \mathcal{U}$. For an illustration, we choose the parameters as $c_S = c_V = 1$, $\Lambda_{\mathcal{U}} = 1 \text{ TeV}$, $d_{\mathcal{U}} = 1.1$ for scalar and $d_{\mathcal{U}} = 2.1$ for vector unparticles. The parameters satisfy the cosmological constraints. For the vector unparticle contribution, the rates of two-body and three-body decays are

$$\begin{aligned} \Gamma_V(\nu_\mu \rightarrow \nu_e + \mathcal{U}) &= 4 \times 10^{-34} \text{ eV}, \\ \Gamma_V(\nu_\mu \rightarrow \nu_e + \bar{\nu}_e + \nu_e) &= 1 \times 10^{-66} \text{ eV}. \end{aligned} \quad (25)$$

The three-body decay rate is more than 30 orders smaller than that of the two-body case. The tiny ratio is due to the very weak coupling between unparticle and neutrinos (and there are two such vertices for the process, see Fig. 2.) and small neutrino mass. This observation is analogous to the process of $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$ where the decay rate is proportional to G_F^2 and m_μ^5 . If we only use the three-body decay, a life time of neutrino which is so long that muon neutrino will never decay and the bump observed in the experiment cannot be explained by the neutrino decays at all. Thus, we can safely neglect the contributions from the three-body decays and approximate $\Gamma_\nu = \Gamma(\nu_\mu \rightarrow \nu_e + \mathcal{U})$.

Secondly, we discuss the constraints of unparticle parameters from neutrino decays. As discussed above, the MiniBooNE experiments put a bound for neutrino life time in the rest frame $\tau_\nu/m_\nu \sim 10^{-4}$ s/eV. We take this bound for our analysis and discuss three possibilities. In order for the illustration, we are restricted in the case of vector unparticle. (1) We fix $d_{\mathcal{U}} = 2.1$, $\Lambda_{\mathcal{U}} = 1$ TeV, and constrain c_V by $c_V > 10^{11}$. The coupling constants are found to be much larger than the order 1. (2) We fix $c_S = c_V = 1$, $\Lambda_{\mathcal{U}} = 1$ TeV, and constrain $0 < d_{\mathcal{U}} - 2 < 10^{-7}$. The scale dimension will be nearly equal to 2. (3) We fix $c_S = c_V = 1$, $d_{\mathcal{U}} = 2.1$, and constrain $\Lambda_{\mathcal{U}} < 10^{-9}$ TeV which is obviously impossible. Thus, if neutrino decays as suggested, the unparticle parameters have to fall into a very unnatural space. On the opposite side, if the unparticle parameters are chosen in a reasonable space, the neutrino life time will be so long that they will not decay when flying over the distance $L \sim 500$ m in the MiniBooNE.

Thirdly, we discuss the relative energy spectrum of final electron neutrino. Fig. 3 plots the initial ν_μ energy spectrum. The distribution is a quasi-Gaussian function, which peaks around 700 MeV. If the muon neutrino decays to electron neutrino and a conventional particle, the energy spectrum of electron neutrino has the same distribution as that the muon neutrino beam. From the data of excess events plotted Fig. 4, obviously, the ν_e energy spectrum is not consistent with the data, no matter for low energy or high energy. The experimental data show that the excess of ν_e events is a decreasing function rather than a Gaussian distribution in the energy range $0.3 < E_{\nu_e} < 1.0$ GeV. This excludes the neutrino decays with conventional particles. Unparticle is not a regular particle and has no a fixed mass. The energy distribution of ν_e with unparticle being in the final state is different from the initial ν_μ spectrum and the final energy distribution of ν_e depends on both effects. Combining the ν_μ energy spectrum (energy spreading of the muon neutrino beam) and the differential ratios of neutrino decay given in Eqs. (18,19), the final electron neutrino energy distribution is depicted in Figs. 4 and 5 for scalar and vector unparticles, respectively. Within the non-oscillation region, i.e., at low energy $0.3 < E_{\nu_e} < 0.45$ GeV, the theory prediction is well consistent with the experimental data. For the energy range $0.5 < E_{\nu_e} < 1.05$ GeV, the theory prediction is slightly larger than the data. It is noted that in the above figures, only the relative size is estimated, the absolute magnitude is very small.

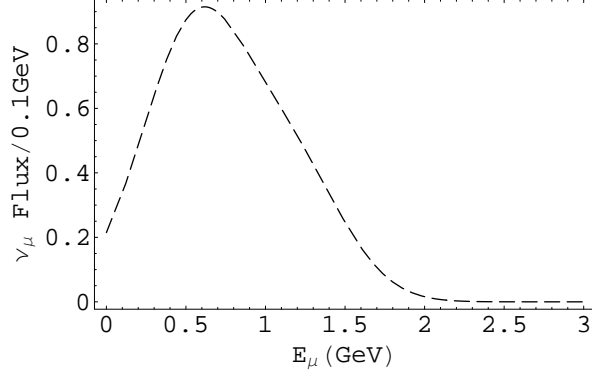


FIG. 3: The energy spectrum for the ν_μ beam in the laboratory frame.

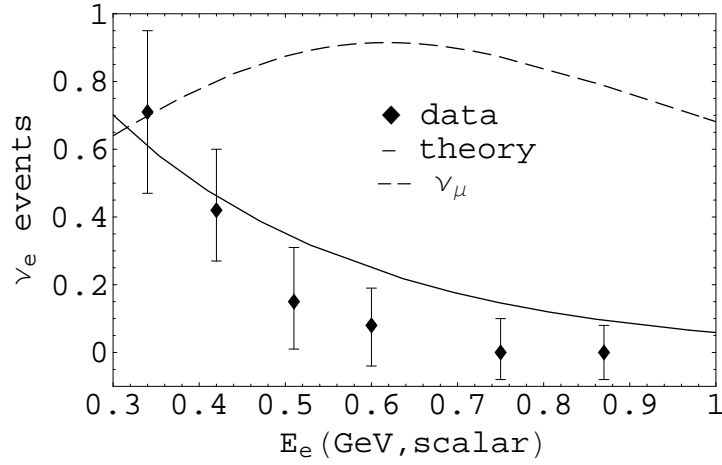


FIG. 4: The energy spectrum for the decay of $\nu_\mu \rightarrow \nu_e + \mathcal{U}$ with scalar unparticle in the laboratory frame where $d_{\mathcal{U}} = 1.1$, $\Lambda_{\mathcal{U}} = 1$ TeV and $c_S = c_V = 1$.

IV. DISCUSSIONS AND CONCLUSIONS

Motivated by the new measurement of the MiniBooNE Collaboration, which observed an excess of electron-like events at low energy, it motivates us to search for a possible mechanism beyond the SM to explain this phenomenon, so the first idea which hits our mind is that ν_2 might decay into ν_1 accompanied by some other very light products. We should testify if this scenario can produce results which are theoretically plausible and can explain the data.

There may be several possible modes, the first one is $\nu_2 \rightarrow \nu_1 + \bar{\nu}_1 + \nu_1$ which is a three-body decay, the second one is $\nu_2 \rightarrow \nu_1 + a$ where a is a single boson particle, for example an axion etc., and the third one is $\nu_2 \rightarrow \nu_1 + \mathcal{U}$ where \mathcal{U} represents an unparticle. All the three possibilities cannot be realized in the framework of the SM, so new physics beyond the SM is necessary. The first one was numerically estimated in this work and our results

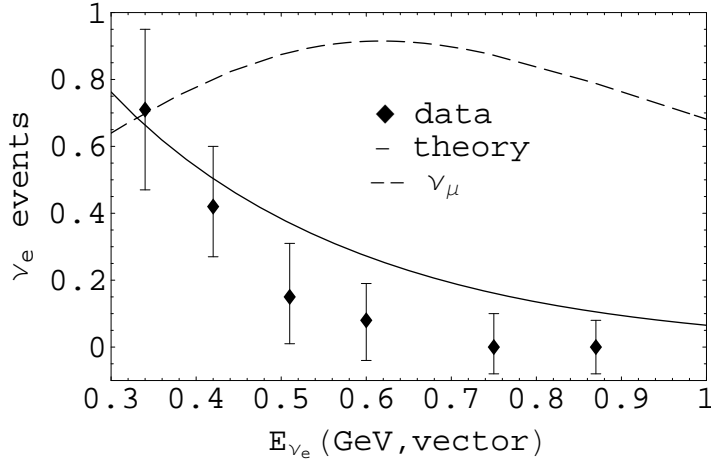


FIG. 5: The energy spectrum for the decay of $\nu_{\mu} \rightarrow \nu_e + \mathcal{U}$ with vector unparticle where $d_{\mathcal{U}} = 2.05$, $\Lambda_{\mathcal{U}} = 1$ TeV and $c_S = c_V = 1$.

indicate that the decay rate determined by the three-body decay mode is too small and is ruled out immediately. The second mode is a two-body decay, therefore the spectrum of the electron neutrinos is discrete and it is not consistent with the measurement of the MiniBooNE. Even though we consider the energy spreading of the incident muon neutrino beam, the shape of the resultant electron neutrino bump cannot be well understood in this scenario. Therefore the third candidate is the most favorable. In this work, we work out the formulations of neutrino decays within the framework of unparticle physics. The formulations in the laboratory frame are given for the first time.

The smallness of the decay width given by our numerical results indicates that the unparticle scenario may not explain the excess of electron neutrinos at low energy. The life time predicted in the unparticle model is qualitatively consistent with the cosmological constraint [43] which is about 10^{17} sec. By eq. (1), we know the suppression of $\exp(-t/10^{17})$ with $t \sim 10^{-7}$ sec., would kill any possibility of observing a decay event. The reasons are: (1) very tiny neutrino mass (2) very weak interactions between the unparticle and neutrino. Since our numerical results are consistent with the cosmology constraints and the results by other authors [26], we can be convinced that the calculation is right, but the proposal does not work here.

Thus in this work, we definitely obtain a negative conclusion that the peak of electron-neutrino at lower energy observed by the MiniBooNE collaboration cannot be explained by the neutrino decay. On other aspect, the phenomenon is there and demands theoretical explanations, so that we propose another scenario which might overcome the aforementioned restrictions which forbid the appearance of electron neutrinos to appear at low energy for the MiniBooNE experiments. We will present the scenario in our next work.

Acknowledgments

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